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# Toward an experimental determination of possible vacuum regeneration for neutral flavoured mesons

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**Abstract.** The usual phenomenology of the complex formed by a neutral flavoured meson $M^0$ and its antiparticle $\bar{M}^0$ assumes the absence of vacuum regeneration in this complex. We propose experiments for determining the two amplitudes of (possibly non-zero) vacuum regeneration: (i) a comparison of the time dependence of decays of the $M^0$ and $\bar{M}^0$ into a channel which could be a $CP$-eigenstate (e.g., $\pi^+\pi^-$ or $\pi^0\pi^0\pi^0$), or a general channel like $\pi l\nu$; (ii) a measurement of the ratio of the time-dependent transmutations, $(M^0 \to \bar{M}^0)$ and $(\bar{M}^0 \to M^0)$; (iii) a measurement of the ratio of the time-dependent probabilities for the production of $|M^0M^0\rangle$ and $|\bar{M}^0\bar{M}^0\rangle$ states, starting with a $C$-odd correlated $|M^0\bar{M}^0\rangle$ state like the $\phi$-meson. The proposed experiments are required to be as accurate as those for the known $CP$-violation effects in the $(M^0, \bar{M}^0)$ complex.

## 1 Introduction

The complex formed by a neutral flavoured meson $M^0$ (i.e., $K^0, D^0, B^0_d, B^0_s$) and its antiparticle $\bar{M}^0$ is known to be suitable for studying violations of the discrete symmetries $CP$, $T$ and $CPT$, and for looking for new physics beyond the standard model; for recent reviews, see [1, 2]. Because of the importance of the conclusions arising out of these investigations, it is highly desirable to use, as a basis, a phenomenology which is as model independent as feasible. The commonly employed phenomenology is based on the Weisskopf–Wigner approximation (abbreviated henceforth as WWA); for a review, see [3]. Being an approximation, the WWA cannot provide an exact theory; see, e.g., [4, 5]. In the WWA, the two flavour states $|M^0\rangle$ and $|\bar{M}^0\rangle$ are linearly superposed to form two "mass eigenstates" which propagate, in time, independently of each other. In other words, when left to itself, one mass eigenstate does not convert into the other, i.e., there is "lack of vacuum regeneration" (abbreviated henceforth as LVR). The LVR property of the WWA leads to a lot of simplification. Some tests of the LVR are known [5–10].

Using general principles, Khalfin showed [5], through his "Main Theorem, part (A)", that the LVR cannot be correct; see also [11]. He predicted a "new $CP$-violation effect" arising from violations of the LVR. However, his estimate of this new effect was theoretically found [12] to be too large by many orders of magnitude.

In the final analysis, the true size of possible LVR violations must be determined experimentally. The purpose of the present paper is to propose experiments for determining the two (complex and time-dependent) amplitudes of vacuum regeneration. While our discussion is explicitly meant for the neutral kaon choice for $M^0$, our considerations are easily generalised to the other choices. Needless to say, the present data are consistent with the phenomenology which incorporates the LVR.

The plan of this paper is as follows. In Sect. 2 is given the formalism for describing the LVR and its possible violations. Section 3 reviews the existing tests of the LVR. Section 4 describes the experiments we propose for determining the two amplitudes of vacuum regeneration; these correspond to the tests of Sect. 3. A summary and some discussion of our results is given in the last section. We shall allow violation of $CP$-, $T$- and $CPT$-invariances to occur throughout.

## 2 The formalism for describing violations of the LVR

As mentioned above, we choose $M^0 = K^0$ for our explicit discussion; the generalisation to other choices for $M^0$ is straightforward. Let us superpose the flavour states $|K^0\rangle$ and $|\bar{K}^0\rangle$ to form two independent normalised states $|K_{1,2}\rangle$ by using the complex constants $p_{\mathrm{S,L}}$ and $q_{\mathrm{S,L}}$:

$$\begin{aligned} |K_1\rangle &= p_{\mathrm{S}}|K^0\rangle + q_{\mathrm{S}}|\bar{K}^0\rangle, \\ |K_2\rangle &= p_{\mathrm{L}}|K^0\rangle - q_{\mathrm{L}}|\bar{K}^0\rangle, \end{aligned} \tag{1}$$

with

$$|p_{\mathrm{S}}|^2 + |q_{\mathrm{S}}|^2 = 1 = |p_{\mathrm{L}}|^2 + |q_{\mathrm{L}}|^2 \quad . \tag{2}$$

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The above transformation is invertible:

$$|K^0\rangle = (q_L|K_1\rangle + q_S|K_2\rangle)/d,$$
$$|\bar{K}^0\rangle = (p_L|K_1\rangle - p_S|K_2\rangle)/d, \tag{3}$$

where $d = p_S q_L + p_L q_S$. If we define the general probability amplitudes for the time-dependent transitions $|K^0\rangle \to |K^0\rangle, |K^0\rangle \to |\bar{K}^0\rangle, |\bar{K}^0\rangle \to |K^0\rangle$ and $|\bar{K}^0\rangle \to |\bar{K}^0\rangle$ respectively as $a(t), b(t), \bar{b}(t)$ and $\bar{a}(t)$, where $t$ is the proper time, the general time dependence of the states $|K_{1,2}\rangle$ is given by

$$|K_1\rangle \to \Theta_S(t)|K_1\rangle + D_{SL}(t)|K_2\rangle,$$
$$|K_2\rangle \to D_{LS}(t)|K_1\rangle + \Theta_L(t)|K_2\rangle, \tag{4}$$

where the functions $\Theta_{S,L}, D_{SL}$ and $D_{LS}$ are given by [5, 10]

$$\Theta_S + \Theta_L = a + \bar{a}, \tag{5a}$$
$$\Theta_S - \Theta_L = [(a-\bar{a})(p_S q_L - q_S p_L) + 2b p_S p_L + 2\bar{b} q_S q_L]/d, \tag{5b}$$
$$D_{SL} = [(a-\bar{a})p_S q_S - b p_S p_S + \bar{b} q_S q_S]/d, \tag{5c}$$
$$D_{LS} = [(a-\bar{a})p_L q_L + b p_L p_L - \bar{b} q_L q_L]/d. \tag{5d}$$

Because of (2), only six real parameters determine $p_{S,L}$ and $q_{S,L}$. Out of these, only three are significant [1–3]. We shall choose these by using the Eberhard convention [13]

$$p_S = e^{i\theta/2}\mathrm{Cos}\alpha_S, \quad q_S = e^{-i\theta/2}\mathrm{Sin}\alpha_S,$$
$$p_L = e^{-i\theta/2}\mathrm{Cos}\alpha_L, \quad q_L = e^{i\theta/2}\mathrm{Sin}\alpha_L, \tag{6}$$

where the real parameters $\alpha_{S,L}$ and $\theta$ lie in the ranges

$$0 \le \alpha_{S,L} \le \frac{\pi}{2}, \quad -\frac{\pi}{2} \le \theta \le +\frac{\pi}{2}. \tag{7}$$

In view of the known smallness of the $CP$-violation, one would like the experimentally interesting superpositions $|K_{1,2}\rangle$ to depart only slightly from the $CP$-eigenstates:

$$|K_\pm\rangle = \frac{1}{\sqrt{2}}[|K^0\rangle \pm |\bar{K}^0\rangle], \tag{8}$$

where we have used the definitions

$$CP|K^0\rangle = |\bar{K}^0\rangle, \quad CP|\bar{K}^0\rangle = |K^0\rangle. \tag{9}$$

In this situation one may retain the $CP$-violating parameters implied in (6) up to only the first order. Then, one finds

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left[\left(1 + \frac{\sigma - \delta + i\theta}{2}\right)|K^0\rangle + \left(1 - \frac{\sigma - \delta + i\theta}{2}\right)|\bar{K}^0\rangle\right]$$
$$|K_2\rangle = \frac{1}{\sqrt{2}}\left[\left(1 + \frac{\sigma + \delta - i\theta}{2}\right)|K^0\rangle - \left(1 - \frac{\sigma + \delta - i\theta}{2}\right)|\bar{K}^0\rangle\right] \tag{10}$$

where $\theta$ and the two combinations

$$\sigma = \frac{\pi}{2} - (\alpha_S + \alpha_L), \quad \delta = (\alpha_S - \alpha_L) \tag{11}$$

would vanish in the limit of $CP$-invariance; we shall retain $\theta, \sigma$ and $\delta$ up to only the first order.

Now let us consider the WWA. Here, the characteristic property is the introduction of two states $|K_{S,L}\rangle$ which propagate, in time, independently of each other; the "vacuum regeneration" transitions $|K_S\rangle \to |K_L\rangle$ and $|K_L\rangle \to |K_S\rangle$ are absent. In terms of (4), the special cases $|K_{S,L}\rangle$ of, respectively, $|K_{1,2}\rangle$ are defined through the time dependence

$$|K_S\rangle \to \Theta_S(t)|K_S\rangle,$$
$$|K_L\rangle \to \Theta_L(t)|K_L\rangle, \tag{12}$$

whereby the states $|K_{S,L}\rangle$ become "mass eigenstates" with complex masses $\lambda_{S,L}$:

$$\Theta_{S,L} = \exp(-it\lambda_{S,L}),$$
$$\lambda_{S,L} = m_{S,L} - \frac{i}{2}\gamma_{S,L}, \tag{13}$$

where $m_{S,L}$ and $\gamma_{S,L}$ are the usual masses and decay widths for the propagation states $|K_{S,L}\rangle$. In the WWA, for the states $|K_{S,L}\rangle$,

$$D_{SL} = 0 = D_{LS}, \tag{14}$$

which is *the* LVR, with its characteristic predictions [14]

$$\bar{a} - a = \beta b, \tag{15a}$$
$$\bar{b} = \alpha b, \tag{15b}$$
$$b = (\Theta_S - \Theta_L)q_S q_L/d, \tag{15c}$$
$$\alpha = \frac{p_S p_L}{q_S q_L}, \quad \beta = \left(\frac{p_L}{q_L} - \frac{p_S}{q_S}\right). \tag{15d}$$

Our purpose is to propose experiments for determining the amplitudes $D_{SL}$ and $D_{LS}$ of vacuum regeneration. Without the LVR, the four independent amplitudes are $(\Theta_S \pm \Theta_L), D_{SL}$ and $D_{LS}$, equivalent to the counting $a, b, \bar{b}$ and $\bar{a}$; see (5). With the LVR, one has only $(\Theta_S \pm \Theta_L)$ which suffice because of (5a) and (15). For the mixing parameters, there is no formal change in going from $|K_{1,2}\rangle$ to $|K_{S,L}\rangle$; (1)–(3), (6), (7), (10) and (11) written for the $|K_{1,2}\rangle$ apply also to the $|K_{S,L}\rangle$ case. Then, the $|K_S\rangle(|K_L\rangle)$ is the short-lived (long-lived) state which is predominantly $CP$-even ($CP$-odd).

It is useful to write $D_{SL}$ and $D_{LS}$ in the Eberhard convention:

$$D_{SL} - D_{LS} = (\bar{b} - b) - \sigma(b + \bar{b}), \tag{16a}$$
$$D_{SL} + D_{LS} = (a - \bar{a}) + (\delta - i\theta)(b + \bar{b}). \tag{16b}$$

Because of (9), $CP$-invariance directly gives

$$\bar{a} = a, \quad \bar{b} = b. \tag{17}$$

Also, $\theta, \sigma$ and $\delta$ parameterise $CP$-violation in mixing; see (8)–(11). Therefore, (16) show that the amplitudes $D_{SL}$

and $D_{\mathrm{LS}}$ are of first order in $CP$-violation. Henceforth, all $CP$-violations will be retained up to only the first order. If $D_{\mathrm{SL}}$ and $D_{\mathrm{LS}}$ vanish due to LVR, one gets the usual WWA results of (15a), (15b) and (15d), stated in the Eberhard convention as

$$\bar{b} = (1+2\sigma)b, \tag{18a}$$

$$(\bar{a}-a) = 2(\delta - \mathrm{i}\theta)b, \tag{18b}$$

since now,

$$\alpha = 1+2\sigma, \quad \beta = 2(\delta-\mathrm{i}\theta). \tag{19}$$

## 3 Some existing tests of the LVR

We shall list existing tests for the situation when the final state arises from the flavour states $|K^0\rangle$ and $|\bar{K}^0\rangle$, and, secondly, when the final state consists of physical decay channels (e.g. $\pi^+\pi^-, \pi^0\pi^0\pi^0$ and $\pi l\nu$). In both cases, we shall take the initial state to be either the flavour states $K^0$ and $\bar{K}^0$ themselves (here, only one time is involved), or the $C$-odd state (e.g., the $\phi$-meson):

$$|-\rangle = \frac{1}{\sqrt{2}}|K^0\bar{K}^0 - \bar{K}^0K^0\rangle, \tag{20}$$

wherein two time variables will be involved.

Firstly, the one-time case. The $t$ independence of the ratio

$$\left|\bar{b}/b\right|^2 = |\alpha|^2 \tag{21}$$

following from (15b) is known [7, 8, 15] as a test of reciprocity. Correspondingly, the bounds [16]

$$-|\beta| \le \left|\frac{\bar{a}}{b}\right| - \left|\frac{a}{b}\right| \le +|\beta| \tag{22}$$

follow from (15a); however, these bounds are not useful equality-type tests. Coming to the situation when the physical channel $k$ forms the final state, and the flavour states $K^0$ and $\bar{K}^0$ form the initial state, one writes the decay rate into $k$ as

$$R^k_\pm(t) = a^k_\pm\left|\Theta_{\mathrm{S}}(t)\right|^2 + b^k_\pm\left|\Theta_{\mathrm{L}}(t)\right|^2 + 2\mathrm{Re}(c^k_\pm\Theta^*_{\mathrm{L}}\Theta_{\mathrm{S}}), \tag{23}$$

where the subscripts $\pm$ refer to the initial $K^0$ and $\bar{K}^0$ states respectively. Then LVR gives [6, 17]

$$a^k_+/a^k_- = 1-2(\sigma+\delta), \tag{24a}$$

$$b^k_+/b^k_- = 1-2(\sigma-\delta), \tag{24b}$$

$$c^k_+/c^k_- = -(1-2\sigma+2\mathrm{i}\theta), \tag{24c}$$

which holds for any general channel $k$ like $\pi l\nu$. In case one has a $CP$-even decay channel (e.g., $\pi^+\pi^-$), (23) predicts the corresponding decay rates to have the time dependences

$$R^{\mathrm{even}}_\pm \to \left|\Theta_{\mathrm{S}}\left(1\mp\frac{\sigma+\delta-\mathrm{i}\theta}{2}\right)\pm\eta\Theta_{\mathrm{L}}\right|^2, \tag{25a}$$

where $\eta$ is the $CP$-violating ratio of the constant amplitudes of $K_{\mathrm{L}}$ and $K_{\mathrm{S}}$ decays into the chosen $CP$-even channel; this gives the $t$ dependence

$$(R^{\mathrm{even}}_+ - R^{\mathrm{even}}_-) \to \mathrm{Re}(\eta\Theta_{\mathrm{L}}\Theta^*_{\mathrm{S}}) - \frac{1}{2}(\sigma+\delta)\left|\Theta_{\mathrm{S}}\right|^2. \tag{25b}$$

If the decay channel chosen is $CP$-odd (e.g., $\pi^0\pi^0\pi^0$), one gets the corresponding results

$$R^{\mathrm{odd}}_\pm \to \left|\Theta_{\mathrm{S}}\chi \pm \Theta_{\mathrm{L}}\left(1\mp\frac{\sigma-\delta+\mathrm{i}\theta}{2}\right)\right|^2, \tag{26a}$$

$$(R^{\mathrm{odd}}_+ - R^{\mathrm{odd}}_-) \to \mathrm{Re}(\Theta^*_s\chi^*\Theta_{\mathrm{L}}) - \left(\frac{\sigma-\delta}{2}\right)\left|\Theta_{\mathrm{L}}\right|^2, \tag{26b}$$

where $\chi$ is the $CP$-violating ratio of the constant amplitudes of $K_{\mathrm{S}}$ and $K_{\mathrm{L}}$ decays into the chosen $CP$-odd channel.

For decays of the state $|-\rangle$ at rest, we consider the probability asymmetry [9, 18]

$$A(t_1,t_2) = \frac{P(t_1,t_2)-\bar{P}(t_1,t_2)}{P(t_1,t_2)+\bar{P}(t_1,t_2)}, \tag{27}$$

where $P$ is the probability for detection of the first $K^0$ at time $t_1$ and the second $K^0$ at time $t_2$, and $\bar{P}$ is the probability for detection of the first $\bar{K}^0$ at time $t_1$ and the second $\bar{K}^0$ at time $t_2$. Using the general formulas

$$P(t_1,t_2) = \frac{1}{2}\left|a(t_1)\bar{b}(t_2)-\bar{b}(t_1)a(t_2)\right|^2, \tag{28a}$$

$$\bar{P}(t_1,t_2) = \frac{1}{2}\left|b(t_1)\bar{a}(t_2)-\bar{a}(t_1)b(t_2)\right|^2, \tag{28b}$$

one finds the LVR test [5, 9, 18] following from (15a) and (15b):

$$A(t_1,t_2) = \frac{|\alpha|^2-1}{|\alpha|^2+1}\,\frac{\text{Eberhard}}{\text{convention}} \to 2\sigma, \tag{29}$$

which is independent of both $t_1$ and $t_2$, and equals the asymmetry

$$B(t) = \frac{\left|\bar{b}\right|^2-|b|^2}{\left|\bar{b}\right|^2+|b|^2} = \frac{|\alpha|^2-1}{|\alpha|^2+1} \tag{30}$$

of the one-time case; see (21). For decays of the state $|-\rangle$ at rest into physical channels $f, g$ at times $t_1$ and $t_2$ respectively, one writes, using [10] the closed nature of the $[(K^0,\bar{K}^0)\leftrightarrow(K_{\mathrm{S}},K_{\mathrm{L}})]$ system the decay rate

$$\begin{aligned} R(f,t_1;g,t_2) = \frac{1}{2}\Big|&(a(t_1)\bar{b}(t_2)-\bar{b}(t_1)a(t_2))A_fA_g \\ &+ (b(t_1)\bar{a}(t_2)-\bar{a}(t_1)b(t_2))\bar{A}_f\bar{A}_g \\ &+ \Big(a(t_1)\bar{a}(t_2)+b(t_1)\bar{b}(t_2) \\ &- \bar{a}(t_1)a(t_2)-\bar{b}(t_1)b(t_2)\Big)\frac{1}{2}(A_f\bar{A}_g+\bar{A}_fA_g) \\ &+ \Big(a(t_1)\bar{a}(t_2)-b(t_1)\bar{b}(t_2) \\ &+ \bar{a}(t_1)a(t_2)-\bar{b}(t_1)b(t_2)\Big)\frac{1}{2}(A_f\bar{A}_g-\bar{A}_fA_g)\Big|^2, \end{aligned} \tag{31}$$

which leads to the factorised time dependence [10]

$$R(f,t_1;g,t_2) = \frac{1}{2}\left|a(t_1)b(t_2) - b(t_1)a(t_2)\right|^2 \times \left|\alpha A_f A_g - \bar{A}_f\bar{A}_g - \beta A_f \bar{A}_g\right|^2 \quad (32)$$

for the situation in which the condition

$$A_f\bar{A}_g - \bar{A}_f A_g = 0 \quad (33)$$

holds, if the LVR relations of (15a) and (15b) are used; here the transition amplitudes are defined by

$$A_{f,g} = \langle f,g\,|T|\,K^0\rangle, \quad \bar{A}_{f,g} = \langle f,g\,|T|\,\bar{K}^0\rangle. \quad (34)$$

The LVR test is the universality of the dependence of $R(f,t_1;g,t_2)$ on $t_1$ and $t_2$, as given in (32), for every $f$ and $g$ satisfying (33).

## 4 Experiments for determining the amplitudes $D_{\mathrm{SL}}$ and $D_{\mathrm{LS}}$ of vacuum regeneration

We now want to see how the tests of Sect. 3 are modified if non-zero vacuum regeneration amplitudes are allowed. The analysis is based on (5) and (16). One obtains

$$\bar{b}/b = \alpha + H(t), \quad (35a)$$

$$(\bar{a}-a)/b = \beta + F(t), \quad (35b)$$

where

$$H(t) = 2(D_{\mathrm{LS}} - D_{\mathrm{SL}})/(\Theta_{\mathrm{L}} - \Theta_{\mathrm{S}}),$$
$$F(t) = 2(D_{\mathrm{LS}} + D_{\mathrm{SL}})/(\Theta_{\mathrm{L}} - \Theta_{\mathrm{S}}),$$

instead of (15a) and (15b). Thus the LVR results of (21) and (22) are replaced by

$$\left|\bar{b}/b\right|^2 = |\alpha|^2 + 2\mathrm{Re}H(t), \quad (36a)$$

$$-\left|\beta + F(t)\right| \le \left|\frac{\bar{a}}{b}\right| - \left|\frac{a}{b}\right| \le +\left|\beta + F(t)\right|. \quad (36b)$$

Therefore, some information on $(D_{\mathrm{SL}} - D_{\mathrm{LS}})$, in terms of $(\Theta_{\mathrm{S}} - \Theta_{\mathrm{L}})$, may be obtained from experimental data on $(\bar{b}/b)^2$, but the bounds (36b) are not equally useful. The new point is the time dependence of the ratio $\left|\bar{b}/b\right|^2$, and of the bounds (36b). Consequently, $\left|\bar{b}/b\right|^2$ can have its reciprocity value $(=1)$ at a certain time $t$ even though the structural constant $\sigma$ is non-zero, in contrast to the prediction from (19) and (21). Similarly, $|\bar{a}/a|$ can have its $CPT$-invariance value $(=1)$ even though $\beta$ is non-zero; contrast this with the bounds of (22).

For the tests of (24), one now obtains

$$a_+^k/a_-^k = 1 - 2(\sigma+\delta) + 4\mathrm{Re}(D_{\mathrm{LS}}/\Theta_{\mathrm{S}}), \quad (37a)$$

$$b_+^k/b_-^k = 1 - 2(\sigma-\delta) + 4\mathrm{Re}(D_{\mathrm{SL}}/\Theta_{\mathrm{L}}), \quad (37b)$$

$$c_+^k/c_-^k = -\left[1 - 2\sigma + 2\mathrm{i}\theta + 2\left(\frac{D_{\mathrm{LS}}}{\Theta_{\mathrm{S}}} + \left(\frac{D_{\mathrm{SL}}}{\Theta_{\mathrm{L}}}\right)^*\right)\right], \quad (37c)$$

where the new point is the time dependence on the right-hand sides of (37) which show how some information on $D_{\mathrm{LS}}$ (in terms of $\Theta_{\mathrm{S}}$) and $D_{\mathrm{SL}}$ (in terms of $\Theta_{\mathrm{L}}$ ) may be obtained from experimental data on the ratios $(a_+^k/a_-^k)$, $(b_+^k/b_-^k)$ and $(c_+^k/c_-^k)$ for any general $k$ (e.g., $\pi l\nu$) which is not a $CP$-eigenstate. It is worth noting that the coefficients $a_\pm^k, b_\pm^k$ and $c_\pm^k$ of (23) would be constants (as in the LVR framework) if there is no $CP$-violation, but their (supposedly small) $CP$-violating parts now have a time dependence due to $D_{\mathrm{SL}}$ and $D_{\mathrm{LS}}$.

Now consider the "charge asymmetry $\mathcal{A}_{\mathrm{L}}$ (in the notation of [17]) in the semileptonic decays of a $K_{\mathrm{L}}$ beam". A priori, this asymmetry is not well defined because vacuum regeneration does not allow one to think of a $K_{\mathrm{L}}$ beam. However, if one defines $\mathcal{A}_{\mathrm{L}}$ in terms of the $b_\pm^k$ (now, time dependent) of (23) as (see, e.g., [17])

$$\mathcal{A}_{\mathrm{L}} = (b_+^+ - b_+^-)/(b_+^+ + b_+^-),$$

where the superscript indicates the leptonic charge in the $\pi l\nu$ decay channel, it is easy to see that the new $\mathcal{A}_{\mathrm{L}}$ derived from an arbitrary initial incoherent mixture of $K^0$ and $\bar{K}^0$ beams is the same as that derived from an initial $K^0$ beam, as was true also without vacuum regeneration. This is because the right-hand side of (37b) is independent of the channel $k$, even in the presence of vacuum regeneration. Similarly, the new $\mathcal{A}_{\mathrm{L}}$ has, in form, the old (i.e., without vacuum regeneration) constant value (see, e.g., [17]) in terms of $\sigma, \delta$ and the constant amplitude parameters for $\pi l\nu$ decays, because the relation between the new and the old $b_+^k$ is independent of $k$. Thus $\mathcal{A}_{\mathrm{L}}$ does not provide useful information on $D_{\mathrm{SL}}$ and $D_{\mathrm{LS}}$.

For a $CP$-even decay channel like $\pi^+\pi^-$, the results of (25a) and (25b) are replaced by

$$R_\pm^{\mathrm{even}} \to \left|\Theta_{\mathrm{S}}\left(\left(1 \mp \frac{\sigma+\delta-\mathrm{i}\theta}{2}\right) \pm \frac{D_{\mathrm{LS}}}{\Theta_{\mathrm{S}}}\right) \pm \eta\Theta_{\mathrm{L}}\right|^2, \quad (38a)$$

$$R_+^{\mathrm{even}} - R_-^{\mathrm{even}} \to \mathrm{Re}(\eta\Theta_{\mathrm{L}}\Theta_{\mathrm{S}}^*) - |\Theta_{\mathrm{S}}|^2\left(\frac{\sigma+\delta}{2} - \mathrm{Re}\frac{D_{\mathrm{LS}}}{\Theta_{\mathrm{S}}}\right), \quad (38b)$$

which can give information on $D_{\mathrm{LS}}$ in terms of $\Theta_{\mathrm{S}}$. The $CP$-odd decay channels like $\pi^0\pi^0\pi^0$ give the corresponding information on $D_{\mathrm{SL}}$ in terms of $\Theta_{\mathrm{L}}$ because the results of (26a) and (26b) now get replaced by

$$R_\pm^{\mathrm{odd}} \to \left|\Theta_{\mathrm{S}}\chi \pm \Theta_{\mathrm{L}}\left(1 \mp \frac{\sigma-\delta+\mathrm{i}\theta}{2} \pm \frac{D_{\mathrm{SL}}}{\Theta_{\mathrm{L}}}\right)\right|^2, \quad (39a)$$

$$R_+^{\mathrm{odd}} - R_-^{\mathrm{odd}} \to \mathrm{Re}(\Theta_{\mathrm{S}}^*\chi^*\Theta_{\mathrm{L}}) - |\Theta_{\mathrm{L}}|^2\left(\frac{\sigma-\delta}{2} - \mathrm{Re}\frac{D_{\mathrm{SL}}}{\Theta_{\mathrm{L}}}\right). \quad (39b)$$

In the general situation wherein vacuum regeneration is allowed, we shall express, because of (5), the complete set of flavour-transition amplitudes as $\Theta_{\mathrm{S}}$, $\Theta_{\mathrm{L}}$, $\psi$ and $\rho$ where

$$\psi(t) = D_{\mathrm{SL}}(t)/\Theta_{\mathrm{L}}(t), \quad \rho(t) = D_{\mathrm{LS}}(t)/\Theta_{\mathrm{S}}(t). \quad (40)$$

Then, the amplitude combinations $H$ and $F$ become

$$H(t) = (\rho - \psi)w - (\rho + \psi), \quad (41a)$$
$$F(t) = (\rho + \psi)w - (\rho - \psi), \quad (41b)$$
$$w = (\Theta_{\rm L} + \Theta_{\rm S})/(\Theta_{\rm L} - \Theta_{\rm S}), \quad (41c)$$

where the ratios $\rho$ and $\psi$ represent vacuum regeneration. Then experimental data on $|\bar{b}/b|^2$ of (36a) would involve $\mathrm{Re}(\psi - \rho)$, $\mathrm{Im}(\psi - \rho)$ and $\mathrm{Re}(\psi + \rho)$. Similarly data on $(a_+^k/a_-^k)$ of (37a) would involve $\mathrm{Re}\rho$; data on $(b_+^k/b_-^k)$ of (37b) would involve $\mathrm{Re}\psi$; data on $(c_+^k/c_-^k)$ of (37c) would involve $\mathrm{Re}(\psi+\rho)$ and $\mathrm{Im}(\psi-\rho)$. Data on $(R_+^{\rm even} - R_-^{\rm even})$ of (38b) would involve $\mathrm{Re}\rho$, and data on $(R_+^{\rm odd} - R_-^{\rm odd})$ of (39b) would involve $\mathrm{Re}\psi$. Hence, for a complete determination of $\psi$ and $\rho$, one still needs an observable involving some combination (of $\mathrm{Im}\rho$ and $\mathrm{Im}\psi$) which is independent of $\mathrm{Im}(\psi - \rho)$.

The required combination, $\mathrm{Im}(\psi + \rho)$, occurs in the two-time observable $A(t_1, t_2)$ of (27). One now gets, instead of (29),

$$\begin{aligned} &A(t_1,t_2) - \left(\frac{|\alpha|^2 - 1}{|\alpha|^2 + 1}\right) \\ &= \mathrm{Re}\left[\frac{H(t_1) + H(t_2)}{2}\right. \\ &+ \frac{H(t_1) - H(t_2)}{2}\left(\frac{a(t_1)b(t_2) + b(t_1)a(t_2)}{a(t_2)b(t_1) - a(t_1)b(t_2)}\right) \\ &+ \left.(F(t_1) - F(t_2))\frac{b(t_1)b(t_2)}{a(t_2)b(t_1) - a(t_1)b(t_2)}\right], \end{aligned} \quad (42a)$$

where the right-hand side is the effect of vacuum regeneration. Since $H$ and $F$ are already of first order $CP$-violation, we may write, up to first order in $CP$-violation,

$$\begin{aligned} &A(t_1,t_2) - \left(\frac{|\alpha|^2 - 1}{|\alpha|^2 + 1}\right) \\ &= \mathrm{Re}\left[\frac{H(t_1) + H(t_2)}{2}\right. \\ &+ \frac{1}{2(\Theta_{\rm S}(t_1)\Theta_{\rm L}(t_2) - \Theta_{\rm L}(t_1)\Theta_{\rm S}(t_2))}\Big\{(H(t_1) - H(t_2)) \\ &\times (\Theta_{\rm S}(t_1)\Theta_{\rm S}(t_2) - \Theta_{\rm L}(t_1)\Theta_{\rm L}(t_2)) + (F(t_1) - F(t_2)) \\ &\times (\Theta_{\rm S}(t_1)\Theta_{\rm S}(t_2) + \Theta_{\rm L}(t_1)\Theta_{\rm L}(t_2) \\ &- \left.\Theta_{\rm S}(t_1)\Theta_{\rm L}(t_2) - \Theta_{\rm L}(t_1)\Theta_{\rm S}(t_2))\Big\}\right], \end{aligned} \quad (42b)$$

where, for the zeroth order of $CP$-violation, we have used $a(t) = (1/2)(\Theta_{\rm S}(t) + \Theta_{\rm L}(t))$, and $b(t) = (1/2)(\Theta_{\rm S}(t) - \Theta_{\rm L}(t))$, as can be seen from the inverted version of (5). Now the desired involvement of $\mathrm{Im}(\psi + \rho)$ is seen from (41b) and (42).

We note the following for the observable $A(t_1, t_2)$. As required by quantum mechanics, one must have $t_1 \neq t_2$ in (42) because the two states detected at $t_1$ and $t_2$ are identical in the probability $P$ (and similarly, $\bar{P}$). Because of vacuum regeneration, $A(t_1, t_2)$ acquires a dependence on $t_1$ and $t_2$; see (42). For LVR, $A(t_1, t_2)$ and $B(t)$ were both constant and equal; see (29) and (30). The modified $A$ and $B$ are, in general, neither constant nor equal; see (36a) and (42). A priori, one does not expect the time dependences of $A$ and $B$ to be simply related because $A$ involves two times and both $H$ and $F$, while $B$ involves only one time, and only $H$, but not $F$.

We now come to the modification in the remaining LVR test of Sect. 3, viz. the universality of the time dependence of the two-time-rate $R$ of (32) for detected channels $f$ and $g$ satisfying (33). This universality is now lost. One gets

$$\begin{aligned} R(f,t_1;g,t_2) = \frac{1}{2}\Big|&A_f A_g\Big[\alpha G + (a(t_1)H(t_2)b(t_2) \\ &- H(t_1)b(t_1)a(t_2))\Big] \\ &+ \bar{A}_f\bar{A}_g[-G + b(t_1)b(t_2)(F(t_2) - F(t_1))] \\ &+ A_f\bar{A}_g\Big[\beta G + (H(t_2) - H(t_1))b(t_1)b(t_2) \\ &+ (a(t_1)F(t_2)b(t_2) - F(t_1)b(t_1)a(t_2))\Big]\Big|^2, \\ G = a(t_1)&b(t_2) - b(t_1)a(t_2). \end{aligned} \quad (43)$$

Equation (43) indicates that, in general, the appearance of $H(t_{1,2})$ and $F(t_{1,2})$ does not allow for the universal time dependence $|G|^2$.

Needless to say, the results of Sect. 4 reduce to the corresponding tests of Sect. 3 if the amplitudes of vacuum regeneration would vanish.

## 5 Procedure required and discussion

### 5.1 General summary

Our purpose was to suggest an experimental programme for determining the amplitudes $D_{\rm SL}$ and $D_{\rm LS}$ of vacuum regeneration. For this, we first noted that these amplitudes are of the first order of $CP$-violation; see (16). Then, by considering some existing tests of the LVR and retaining all $CP$-violations up to only the first order, we have defined the required programme which is purely phenomenological. As expected, our programme reproduces the existing tests of the LVR if $D_{\rm SL}$ and $D_{\rm LS}$ are made to vanish. We explicitly considered the case of neutral kaons; the generalization to other flavours is straightforward.

Instead of $D_{\rm SL}$ and $D_{\rm LS}$, our programme aims at the equivalent time-dependent ratios $\psi$ and $\rho$ of (40). The implicit plan is that the chosen data on decays of the neutral flavoured mesons are to be fitted by using $\psi$ and $\rho$ along with the usual full set $S$ of parameters like the constants $\eta, \chi, \sigma, \delta, \theta$ and the functions $\Theta_{\rm S}$ and $\Theta_{\rm L}$; one is not to use the known numerical values of the set $S$ because those values were obtained by starting with the LVR assumption.

The functions $\mathrm{Re}\psi$ and $\mathrm{Re}\rho$ would thus be obtained by the experimental data on the one-time observables $|\bar{b}/b|^2$, $(a_+^k/a_-^k)$, $(b_+^k/b_-^k)$, $(c_+^k/c_-^k)$, $(R_+^{\rm even} - R_-^{\rm even})$, and $(R_+^{\rm odd} -$

$R_-^{\rm odd}$) of, respectively, (36a), (37a), (37b), (37c), (38b) and (39b). The functions Im$\rho$ and Im$\psi$ would be determined by the data on $(c_+^k/c_-^k)$ of (37c) and the two-time ratio $A(t_1, t_2)$ of (42) coupled with (41).

The modifications in the LVR test of (32), due to the non-zero $\psi$ and $\rho$, do not allow a useful prediction to be made because of the unknown amplitude bilinears $A_f A_g, \bar{A}_f \bar{A}_g$ and $A_f \bar{A}_g$.

## 5.2 Practical procedure

### 5.2.1 Functional forms for $\psi$ and $\rho$

One determines $\psi(T)$ and $\rho(T)$ at a particular time $t = T$ by appropriate fits to data, and then varies $T$. The four graphical plots of $\psi$ and $\rho$ as functions of $t$ would thus fulfill the aim of our programme. Possible functional forms may be deduced from these plots; one need not guess the functional forms for starting the data-fitting.

For implementing this point, it will be helpful to consider additional (as compared to Sect. 3) observables wherein the various unknowns occur in combinations different from the ones occurring above. In this context, the observables not involving parameters like $\eta$ and $\chi$ belonging to the decay amplitudes $A_f$ are of special interest. One may, in particular, consider [19] the ($CP$-violating) asymmetry $[(|\bar{a}|^2 - |a|^2)/(|\bar{a}|^2 + |a|^2)]$ and the asymmetry $[(|a|^2 - |b|^2)/(|a|^2 + |b|^2)]$ in analogy to $B(t)$. Similarly, the decays of the correlated $C$-odd state $|-\rangle$ (and the corresponding $C$-even state) to $K^0\bar{K}^0$ and $\bar{K}^0 K^0$ may be considered [16] in addition to $K^0K^0$ and $\bar{K}^0\bar{K}^0$ considered for $A(t_1, t_2)$. For expressing the required amplitudes $a, \bar{a}, b$ and $\bar{b}$ in terms of the interesting parameters, one may use (5a) and

$$\begin{aligned}
\bar{a} - a &= (\Theta_{\rm S} - \Theta_{\rm L})(\delta - {\rm i}\theta) - (D_{\rm SL} + D_{\rm LS}),\\
2b &= (\Theta_{\rm S} - \Theta_{\rm L})(1 - \sigma) - D_{\rm SL} + D_{\rm LS},\\
2\bar{b} &= (\Theta_{\rm S} - \Theta_{\rm L})(1 + \sigma) + D_{\rm SL} - D_{\rm LS}.
\end{aligned}$$

### 5.2.2 Role of $\Theta_{\rm L,S}$

Out of the determinations listed in Sect. 4, that of Re$H$ in (36a) is the only one not requiring knowledge of $\Theta_{\rm L,S}$. The time dependence in the data on $|\bar{b}/b|^2$ would therefore establish non-zero vacuum regeneration, but it still falls short of determining $D_{\rm LS}$ and $D_{\rm SL}$. So one needs $\Theta_{\rm S,L}$.

Retaining $CP$-violations up to only the first order, (5b) easily shows that $(\Theta_{\rm S} - \Theta_{\rm L})$ equals $(b + \bar{b})$, a $CP$-invariant. Of course, $(\Theta_{\rm S} + \Theta_{\rm L})$ is exactly $(a + \bar{a})$, another $CP$-invariant; see (5a). Thus $\Theta_{\rm S}$ and $\Theta_{\rm L}$ are both $CP$-invariant, and, therefore, supposedly large as compared to $D_{\rm SL}$ and $D_{\rm SL}$, which are of first order of the $CP$-violation.

Now compare the coupled set of (4) with the uncoupled set of (12). The percentage error caused in the determination of the large quantities $\Theta_{\rm S,L}$ by dropping couplings arising from the small quantities $D_{\rm SL}$ and $D_{\rm LS}$ is likely to be small. Thus the form of (13) which arises from a solution of (12) may be taken as a good approximation to $\Theta_{\rm S,L}$, for our purposes. To that extent, the final $\psi$ and $\rho$, determined by the use of (13), would not be exact.

Ideally, one should go for a full programme which determines all the parameters (and functions) required for fits to all data on the decays of neutral flavoured mesons; these data include the data considered by us as only a small subset. For such an iterative search programme, useful starting values for those parameters (and functions) which occur in our discussion could be provided by the final values from our programme. The discussion of such a massive programme is outside the scope of the present paper.

## 5.3 Feasibility

The a priori requirement of the accuracy expected from the experiments we propose is indicated by (16); one needs the accuracy relevant for the known $CP$-violation effects which are also of the first order.

Flavour tagging of the final neutral mesons is required for the LVR tests of (21), (22) and (29). For the one-time tests of (21)–(26), one needs flavour tagging for the initial state. The two-time test of (32) which does not require any flavour tagging is, unfortunately, not useful for determining $\psi$ and $\rho$. We merely note the usefulness of the clean CPLEAR procedure [15] for initial state tagging for the neutral kaon case by utilizing the reactions

$$\bar{p}p \to \pi^+ K^- K^0, \pi^- K^+ \bar{K}^0,$$

where the charges of pions and kaons decide the neutral kaon strangeness, assuming only strangeness conservation in the strong interactions. For the final state tagging also, it is advisable to use tagging based on flavour conservation of the strong interactions, in order to avoid assumptions which become necessary if tagging is replaced by decays utilising weak interactions; for a recent emphasis on this issue, see [19]. At present, such final state tagging is not sufficiently accurate [20, 21]. For the heavier flavours, more modern techniques like the “jet-charge method” are being utilized; see [22] for a review.

On the whole, the flavour-tagging techniques available at present are unfortunately not as efficient as the CPLEAR procedure [15]. We hope that the experimental programme suggested in this paper would materialise in the near future.

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